

Modified Brans-Dicke cosmology with matter-scalar field interaction

Georgios Kofinas,^{1,*} Eleftherios Papantonopoulos,^{2,†} and Emmanuel N. Saridakis^{3,4,‡}

¹*Research Group of Geometry, Dynamical Systems and Cosmology,
Department of Information and Communication Systems Engineering
University of the Aegean, Karlovassi 83200, Samos, Greece*

²*Physics Division, National Technical University of Athens, 15780 Zografou Campus, Athens, Greece*

³*CASPER, Physics Department, Baylor University, Waco, TX 76798-7310, USA*

⁴*Instituto de Física, Pontificia Universidad de Católica de Valparaíso, Casilla 4950, Valparaíso, Chile*

We discuss the cosmological implications of an extended Brans-Dicke theory presented recently, in which there is an energy exchange between the scalar field and ordinary matter, determined by the theory. A new mass scale is generated in the theory which modifies the Friedmann equations with field-dependent corrected kinetic terms. In a radiation universe the general solutions are found and there are branches with complete removal of the initial singularity, while at the same time a transient accelerating period can occur within deceleration. Entropy production is also possible in the early universe. In the dust era, late-times acceleration has been found numerically in agreement with the correct behaviour of the density parameters and the dark energy equation of state, while the gravitational constant has only a slight variation over a large redshift interval in agreement with observational bounds.

I. INTRODUCTION

Scalar fields can play an important role in the description of the early universe as well as the late-times cosmic evolution. Scalar-tensor gravitational theories are widely studied as an alternative to General Relativity. Brans-Dicke (BD) gravity [1] is one of the simplest such theories that can be constructed and it is considered a viable alternative to General Relativity, one which respects Mach's principle and weak equivalence principle. Mach's principle states that the property of inertia of material bodies arises from their interactions with the matter distributed in the universe. This theory leads to variations in the Newtonian gravitational constant G and introduces a new dimensionless coupling constant ω . Large values of ω mean a significant contribution from the tensor part, while small values of ω mean an increasing role for the scalar field contribution. General Relativity is recovered in the limit $\omega \rightarrow \infty$. The effective gravitational constant in Brans-Dicke theory is the inverse of the scalar field, namely $G \sim 1/\phi$, however from spherically symmetric solutions it is $G = \frac{4+2\omega}{3+2\omega} \frac{1}{\phi}$.

In modern context, BD theory appears naturally in supergravity models, from string theory at low-energies in the so-called string (Jordan) frame or from the dimensional reduction of Kaluza-Klein theories [2], and this has led to considerable interest in BD gravity. This theory yields the correct Newtonian weak-field limit, but solar system measurements of post-Newtonian corrections require that ω is larger than a few thousands [3]. This is due to the fact that in order to avoid the propagation of the fifth force, the coupling between matter and the massless field ϕ should be suppressed. On the other hand, from cosmological observations ω gets substantially lower values in a model dependent way [4]. The synthesis of light elements during the early Universe [5] provides extra observational constraints upon scalar-tensor theories. There is also the possibility that the gravitational coupling depends on the scale [6], having different value at local and at cosmological scale, and thus it is possible ω to be smaller at cosmological scales giving deviations from General Relativity, while agreement with local tests is preserved.

Many cosmological observations from type Ia supernovae, cosmic microwave background radiation and large scale structure reveal that our universe is undergoing an accelerating expansion. The mysterious component with large enough negative pressure which dominates the dynamics of the universe and drives the cosmic acceleration is called dark energy. The preferred candidate of dark energy is the Einstein's cosmological constant which fits the observations well, but is plagued by the fine-tuning and the cosmic coincidence problems. This recent accelerating period has to be replaced in the past by a decelerating era in order to accommodate for nucleosynthesis in the radiation era and for the formation of galaxies in the matter era.

*Electronic address: gkofinas@aegean.gr

†Electronic address: lpapa@central.ntua.gr

‡Electronic address: Emmanuel.Saridakis@baylor.edu

Most of the models in cosmology consider that the evolutions of dark matter and dark energy occur separately. In [7] it is argued that observational evidences support an interaction between dark energy and dark matter and violation of equivalence principle between baryons and dark matter. Recently there is a raising activity in interacting models of dark matter and dark energy [8], and this flow of energy between the dark matter and the dark energy component can be useful to solve the coincidence problem [9]. Possible mechanisms could alleviate the puzzles arising by the violation of the equivalence principle. In Chameleon mechanism [10] the effective mass of the scalar field can become density dependent, so a large effective mass may be acquired in solar scale hiding local experiments, while at cosmological scales ϕ can be effectively light providing cosmological modifications. An alternative for the resolution of this problem and the recovery of General Relativity in the regions of high energy is the introduction of non-linear self interactions for ϕ through self screening (Vainshtein) mechanisms [11]. Finally, since the validity of the universality of free fall at cosmological scales has not been tested directly, there is the option that the baryonic matter is separately conserved, so as to obey the weak equivalence principle, while dark matter interacts with dark energy.

A series of papers [12]-[19] have found cosmological solutions of Brans-Dicke gravity for radiation, dust or other equations of state or in the presence of a cosmological constant and for all kinds of spatial curvature. Although in cosmology the evolution of the universe generically has a singularity in the past, usually of big bang type as in standard cosmology, in [20] it was argued that a gas of solitonic p -branes treated as a perfect fluid type matter can resolve the initial singularity in the Brans-Dicke theory. In [21] bouncing solutions were found for negative ω in dust-filled universe, while dynamical systems analysis of FRW cosmologies was given in [22]. Brans-Dicke theory is useful in solving some problems of the inflationary scenario with the possibility of extended inflation [23]-[25]. The solution to the “graceful exit” problem of inflation in terms of an extended inflation scenario [24] was first obtained in Brans-Dicke theory without fine tuning, although the value of ω is small in order not to create large anisotropies in the microwave background (see also [26] where the introduction of a potential for the scalar field or a scalar field dependent ω were proposed to solve such problems).

An attractive feature of BD theory is that the scalar field is a fundamental element of the theory which controls the evolution of the gravitational constant and at the same time may possibly form the dark energy. However, it is difficult to succeed acceleration in the standard version of Brans-Dicke theory. There have been found accelerating solutions with $-2 \leq \omega \leq -3/2$ in the matter-dominated universe for a spatially flat universe without cosmological constant or quintessence [27]. Such small values of $|\omega|$ not only are not in agreement with solar system constraints, they also violate the energy conditions on the scalar field and they do not provide a transition to a decelerating era (see also [28]). Spatially closed models with higher values of ω were considered in [29].

Since in the limit $\omega \rightarrow \infty$ the field ϕ becomes fixed and we recover Einstein gravity, this has led to the construction of more general scalar-tensor gravity theories with a self-interacting potential $V(\phi)$, a field or time-dependent ω , or non-minimal couplings [30]. The late-times acceleration of the universe in such models has been studied in [31]-[33], and either fail or succeed to obtain the standard decelerating phase of the universe followed by the recent accelerating period. Although the majority of the models refer to the Jordan frame, in [34] a suitably selected self-interacting potential in Einstein conformal frame was selected and found a class of solutions with accelerated expansion, large positive ω and constant ratio of energy densities of matter and scalar field. There are also other modifications of Brans-Dicke cosmology, such as the introduction of a dissipative cold dark matter fluid [35], a transfer of energy between the dark matter and the Brans-Dicke scalar field [36] (or in the presence of Chaplygin gas [37]), the introduction of a chameleon field [38], holographic dark energy models in the framework of Brans-Dicke cosmologies [39], or other scenarios [40].

Recently, relaxing the standard exact conservation of the matter energy-momentum, but still preserving the simple massless wave equation of motion for the scalar field sourced by the trace of the matter energy-momentum tensor, three general completions of Brans-Dicke gravity were found which are uniquely determined from the consistency of the Bianchi identities [41]. Here, we will focus on the first of these theories, where a new dimensionfull parameter ν appears (which is an integration constant) and for $\nu = 0$ it reduces to the standard Brans-Dicke theory. The energy transfer between ordinary matter and the scalar field defines the non-conservation equation of motion of the matter. The derivation of the theory emerged initially at the level of the field equations. A discussion on the action of the theory was given in [42], where the vacuum action was derived, as well as the full action for special only matter Lagrangians. The matter Lagrangian due to the interaction turns out to be non-minimally coupled even in the Jordan frame, while the issue of the general action of the full theory still remains open.

In the present work we study the cosmological evolution of this theory at early and late times. In the radiation regime we find general exact solutions depending on the parameters of the theory. In the dust period we investigate numerically the cosmic possibilities, where the main mechanism is the energy exchange between dark matter and dark energy predicted by the theory. The scalar field plays at the same time the role of the varying gravitational constant and the role of dark energy. In both cosmic eras we find interesting behaviours of the cosmic evolution which could imitate the history of the universe. The really tempting feature in our approach is the fact that we are strictly focused on the extended Brans-Dicke theory which possesses only well-defined kinetic terms, and we have not added

any ad-hoc structures such as self-interacting potentials or varying functions, contrary to the standard Brans-Dicke cosmology where some extra ingredient is necessary to make the model viable.

The content of the paper is organized as follows: In section II we write down and elaborate the new cosmological equations and integrate the new conservation equation. In section III we integrate the system in the radiation epoch, what leads to the general solutions of the scale factor as a function of the scalar field and discuss their implications at early times. In section IV we numerically integrate the system in the dust regime, we confront against the basic observational data and find the possibility of a recent accelerating era following the decelerating period. Finally, in section V we summarize our results and conclude.

II. COSMOLOGICAL EQUATIONS

The standard Brans-Dicke theory is described by the equations (in units where the velocity of light is set to 1)

$$G^\mu_\nu = \frac{8\pi}{\phi}(T^\mu_\nu + \mathcal{T}^\mu_\nu) \quad (2.1)$$

$$T^\mu_\nu = \frac{2-3\lambda}{16\pi\lambda\phi} \left(\phi^{;\mu}\phi_{;\nu} - \frac{1}{2}\delta^\mu_\nu \phi^{;\rho}\phi_{;\rho} \right) + \frac{1}{8\pi} (\phi^{;\mu}_{;\nu} - \delta^\mu_\nu \square\phi) \quad (2.2)$$

$$\square\phi = 4\pi\lambda\mathcal{T} \quad (2.3)$$

$$\mathcal{T}^\mu_{\nu;\mu} = 0. \quad (2.4)$$

These equations are resulted from a simple action in the Jordan frame of the form

$$S_{BD} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(\phi R - \frac{\omega}{\phi} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right) + \int d^4x \sqrt{-g} L_m, \quad (2.5)$$

where $L_m(g_{\kappa\lambda}, \Psi)$ is the matter Lagrangian depending on scalar fields Ψ . The parameter $\lambda \neq 0$ is dimensionless and is related to the standard Brans-Dicke parameter ω by $\omega = (2-3\lambda)/2\lambda$. The tensor \mathcal{T}^μ_ν is the matter energy-momentum tensor and $\mathcal{T} = \mathcal{T}^\mu_\mu$ is its trace.

The standard Brans-Dicke theory described by equations (2.1)-(2.4) was generalized in [41] by relaxing the strict conservation law (2.4), but still respecting the simple form of the scalar field equation (2.3). As usual, the energy-momentum tensor of the scalar field T^μ_ν was constructed from terms each of which involves two derivatives of one or two scalar fields ϕ , and ϕ itself. Three unique theories were unambiguously determined from consistency at the level of the equations of motion, and here we study the first of these theories which has the following form

$$G^\mu_\nu = \frac{8\pi}{\phi}(T^\mu_\nu + \mathcal{T}^\mu_\nu) \quad (2.6)$$

$$T^\mu_\nu = \frac{\phi}{2\lambda(\nu+8\pi\phi^2)^2} \left\{ 2[(1+\lambda)\nu+4\pi(2-3\lambda)\phi^2] \phi^{;\mu}\phi_{;\nu} - [(1+2\lambda)\nu+4\pi(2-3\lambda)\phi^2] \delta^\mu_\nu \phi^{;\rho}\phi_{;\rho} \right\} + \frac{\phi^2}{\nu+8\pi\phi^2} (\phi^{;\mu}_{;\nu} - \delta^\mu_\nu \square\phi) \quad (2.7)$$

$$\square\phi = 4\pi\lambda\mathcal{T} \quad (2.8)$$

$$\mathcal{T}^\mu_{\nu;\mu} = \frac{\nu}{\phi(\nu+8\pi\phi^2)} \mathcal{T}^\mu_{\nu\phi;\mu}. \quad (2.9)$$

The parameter ν is arbitrary and has dimensions mass to the fourth (it arises as an integration constant from the integration procedure). Its sign and numerical value should be determined experimentally. For $\nu = 0$ the above system of equations reduces to the standard Brans-Dicke theory (2.1)-(2.4). The role of ν is manifest in equation (2.9) and measures the deviation from the exact conservation of matter. The right-hand side of equation (2.6) is consistent with the Bianchi identities, i.e. it is covariantly conserved on-shell, and therefore, the system of equations (2.6)-(2.9) is well-defined.

To investigate the cosmological implications of the above theory, we consider the following spatially homogeneous and isotropic ansatz

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right], \quad (2.10)$$

characterized by the spatial curvature $k = -1, 0, 1$. The temporal gauge choice has been assumed with the lapse function being unity, so t is the cosmic time. The scalar field ϕ respecting the symmetries of the metric (2.10) will be

a function of time, so it is $\phi(t)$. The matter energy-momentum tensor is the one of a perfect fluid $\mathcal{T}_\nu^\mu = \text{diag}(-\rho, p, p, p)$ with $\rho(t)$ being its energy density and $p(t)$ its pressure. Equations (2.6), (2.8) and (2.9) become respectively

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3\phi}\rho - \frac{8\pi\phi}{\nu+8\pi\phi^2}H\dot{\phi} + \frac{4\pi}{3\lambda}\frac{\nu+4\pi(2-3\lambda)\phi^2}{(\nu+8\pi\phi^2)^2}\dot{\phi}^2 \quad (2.11)$$

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -\frac{8\pi}{\phi}\left[p + \frac{\phi}{2\lambda}\frac{(1+2\lambda)\nu+4\pi(2-3\lambda)\phi^2}{(\nu+8\pi\phi^2)^2}\dot{\phi}^2 + \frac{\phi^2}{\nu+8\pi\phi^2}(2H\dot{\phi} + \ddot{\phi})\right] \quad (2.12)$$

$$\ddot{\phi} + 3H\dot{\phi} + 4\pi\lambda(3p-\rho) = 0 \quad (2.13)$$

$$\dot{\rho} + 3H(\rho+p) = \frac{\nu}{\phi(\nu+8\pi\phi^2)}\rho\dot{\phi}. \quad (2.14)$$

The consistency of the above system can be confirmed by checking that the satisfaction of the Bianchi identities is verified. This results to the fact that one of the dynamical equations is redundant and arises from the other equations. We also give for comparison the Brans-Dicke equations which arise from equations (2.11)-(2.14) setting $\nu = 0$

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3\phi}\rho - H\frac{\dot{\phi}}{\phi} + \frac{2-3\lambda}{12\lambda}\frac{\dot{\phi}^2}{\phi^2} \quad (2.15)$$

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -\frac{1}{\phi}\left(8\pi p + \frac{2-3\lambda}{4\lambda\phi}\dot{\phi}^2 + 2H\dot{\phi} + \ddot{\phi}\right) \quad (2.16)$$

$$\ddot{\phi} + 3H\dot{\phi} + 4\pi\lambda(3p-\rho) = 0 \quad (2.17)$$

$$\dot{\rho} + 3H(\rho+p) = 0. \quad (2.18)$$

Equation (2.14) for $p = w\rho$ can be integrated to give

$$\rho = \frac{\rho_*}{a^{3(1+w)}} \frac{|\phi|}{\sqrt{|\nu+8\pi\phi^2|}}, \quad (2.19)$$

where $\rho_* > 0$ is an integration constant. Observe that the energy density evolves in a different way than what would be expected for a self-conserved matter sector. This happens because there is a direct coupling of the matter energy density to the scalar field and this behaviour has important consequences for the cosmological evolution, as we will discuss in the next sections. Since ϕ is related to the gravitational constant through the relation

$$G = \frac{1}{\phi}, \quad (2.20)$$

normally, it should be $\phi > 0$. However, there is a possibility that at the early stages of the universe evolution it is $\phi < 0$, providing an antigravity effect and possibly contributing to the inflationary phase. Therefore, we will include in the following equations both signs of ϕ and restrict our discussion on the properties of the radiation solutions for $\phi > 0$ (of course, ϕ is also positive in the dust universe). Note that a flip from a negative to a positive ϕ seems unnatural since the dynamical evolution of the scalar field would lead to the undesirable situation of going through an infinitely strong gravitational effect.

The relation (2.20) arises from the cosmological considerations of the theory. However, the full correspondence with ϕ would arise from the spherical solutions of the theory and the comparison with the solar system data. However, since in the present work we are interested in investigating the cosmological implications of the theory at hand, the identification (2.20) is adequate. Hence, the only requirement that we should be careful of, is the variance of G to be in agreement with the observational limits, namely $-0.5 \times 10^{-12} \text{ yr}^{-1} \lesssim \frac{\dot{G}}{G}|_0 \lesssim 1.7 \times 10^{-12} \text{ yr}^{-1}$ [43], which in terms of the present value of the Hubble parameter $H_0 \approx (0.71 \pm 0.008) \times 10^{-10} \text{ yr}^{-1}$ [44], becomes $-0.7 \times 10^{-2} \lesssim \frac{\dot{G}}{GH}|_0 \lesssim 2.4 \times 10^{-2}$. This constraint, due to (2.20), becomes $-2.4 \times 10^{-2} \lesssim \frac{\dot{\phi}}{\phi H}|_0 \lesssim 0.7 \times 10^{-2}$. Finally, note that the variation of G is also constrained during the early cosmology by the light elements production in Big Bang Nucleosynthesis (BBN) epoch, with the corresponding bound reading as $\frac{|\delta G|}{G_0 H_0} \lesssim 0.2$ where $\delta G = G_{BBN} - G_0$ [45], which is not a rather strong constraint. No other use of the relation (2.20) will be made in this work, except from the satisfaction of the above bounds on the time variability of the scalar field.

Before we finish this section we can extract the acceleration $\frac{\ddot{a}}{a}$ which will be useful in the following, combining equations (2.11)-(2.13) to be

$$\frac{\ddot{a}}{a} = -\frac{4\pi\rho}{\phi}\left[w + \frac{1}{3} + \lambda(1-3w)\frac{4\pi\phi^2}{\nu+8\pi\phi^2}\right] + \frac{8\pi\phi}{\nu+8\pi\phi^2}H\dot{\phi} - \frac{4\pi}{3\lambda}\frac{(2+3\lambda)\nu+8\pi(2-3\lambda)\phi^2}{(\nu+8\pi\phi^2)^2}\dot{\phi}^2, \quad (2.21)$$

or also

$$\frac{\ddot{a}}{a} = -H^2 - \frac{k}{a^2} + \frac{4\pi(1-3w)\rho}{3\phi} \left(1 - \frac{12\pi\lambda\phi^2}{\nu+8\pi\phi^2}\right) - \frac{4\pi}{3\lambda} \frac{(1+3\lambda)\nu+4\pi(2-3\lambda)\phi^2}{(\nu+8\pi\phi^2)^2} \dot{\phi}^2. \quad (2.22)$$

Finally, note that the case with $\nu+8\pi\phi^2 > 0$ includes two subcases, the first one with $\nu > 0$ and the second with $\nu < 0$, $|\phi| > \sqrt{\frac{|\nu|}{8\pi}}$. The case with $\nu+8\pi\phi^2 < 0$ corresponds to $\nu < 0$, $|\phi| < \sqrt{\frac{|\nu|}{8\pi}}$.

III. EARLY-TIMES COSMOLOGICAL EVOLUTION

We will study the cosmological evolution when the universe is in the radiation era defined by the relativistic equation of state $w = 1/3$. Then, from the scalar field equation (2.13) we see that the scalar field evolution is decoupled from the radiation and this equation is integrated to

$$\dot{\phi}a^3 = c, \quad (3.1)$$

where c is an integration constant. From this equation we get

$$H = \frac{c}{a^4} \frac{da}{d\phi}. \quad (3.2)$$

It is seen from equation (3.1) that ϕ is directly connected with the scale factor, so $\phi(t)$ is either a monotonically increasing or decreasing function in the radiation era. Substituting the expressions (2.19), (3.1) and (3.2) into the Friedmann equation (2.11) we get an equation for $a(\phi)$

$$\left(\frac{da}{d\phi} + \frac{4\pi\phi a}{\nu+8\pi\phi^2}\right)^2 - \frac{4\pi}{3\lambda} \frac{a^2}{\nu+8\pi\phi^2} \left(1 + \frac{2\epsilon\lambda\rho_*}{c^2} a^2 \sqrt{|\nu+8\pi\phi^2|}\right) + \frac{ka^6}{c^2} = 0, \quad (3.3)$$

where $\epsilon = \text{sgn}[\phi(\nu+8\pi\phi^2)]$. Defining

$$\chi = a^2 \sqrt{|\nu+8\pi\phi^2|}, \quad (3.4)$$

we find from equation (3.3)

$$\left(\frac{d\chi}{d\phi}\right)^2 - \frac{16\pi}{3\lambda} \frac{\chi^2}{\nu+8\pi\phi^2} \left(1 + \frac{2\epsilon\lambda\rho_*}{c^2} \chi\right) + \frac{4k}{c^2} \frac{\chi^4}{|\nu+8\pi\phi^2|} = 0. \quad (3.5)$$

Equation (3.5) is a separable equation and can be solved for any k . However, for simplicity we will restrict our interest here to $k = 0$. Then, equation (3.5) becomes

$$\frac{d\chi}{d\phi} = \pm 4\sqrt{\frac{\pi}{3}} \frac{\chi}{\sqrt{|\nu+8\pi\phi^2|}} \sqrt{\frac{\epsilon}{\lambda} + \frac{2\rho_*}{c^2}} \chi, \quad (3.6)$$

where $\frac{\epsilon}{\lambda} + \frac{2\rho_*}{c^2} \chi$ has to be non-negative. There are four cases for the integration of (3.6) concerning the sign of $\nu+8\pi\phi^2$ and the sign of $\epsilon\lambda$. Before presenting the solutions of equation (3.6), we give the expressions for the time integral, the acceleration and the curvature scalar.

The time dependence of ϕ is found from equation (3.1) as

$$t = \frac{1}{c} \int a(\phi)^3 d\phi, \quad (3.7)$$

where a translational integration constant for time has been absorbed. The acceleration $\frac{\ddot{a}}{a}$ is found from (2.21) to be

$$\frac{\ddot{a}}{a} = -\frac{8\pi\rho}{3\phi} + \frac{8\pi\phi}{\nu+8\pi\phi^2} H\dot{\phi} - \frac{4\pi}{3\lambda} \frac{(2+3\lambda)\nu+8\pi(2-3\lambda)\phi^2}{(\nu+8\pi\phi^2)^2} \dot{\phi}^2. \quad (3.8)$$

In the above equation, the quantity $\frac{\rho}{\phi}$ is obtained from equation (2.19); the Hubble parameter H is obtained from equation (3.2), where $\frac{da}{d\phi}$ is found from equation (3.3); finally, the quantity $\dot{\phi}$ is obtained from equation (3.1). Thus,

$\frac{\ddot{a}}{a}$ becomes a function of a, ϕ and after integration of (3.6), it becomes a function of only ϕ . From equation (2.22) we have also the expression

$$\frac{\ddot{a}}{a} = -H^2 - \frac{4\pi}{3\lambda} \frac{(1+3\lambda)\nu + 4\pi(2-3\lambda)\phi^2}{(\nu + 8\pi\phi^2)^2} \dot{\phi}^2, \quad (3.9)$$

from where it is seen when it could be possible to have acceleration, $\ddot{a} > 0$. Using equations (3.1) and (3.6), the acceleration (3.9) can become explicitly a function solely of ϕ given that the equation (3.6) has been integrated

$$-\frac{3}{4\pi c^2} \frac{\chi^3}{\sqrt{|\nu + 8\pi\phi^2|}} \frac{\ddot{a}}{a} = \sqrt{\frac{\epsilon}{\lambda} + \frac{2\rho_*}{c^2}} \chi \left(\sqrt{\frac{\epsilon}{\lambda} + \frac{2\rho_*}{c^2}} \chi \mp \sqrt{\frac{3}{\pi}} \frac{4\pi\epsilon|\phi|}{\sqrt{|\nu + 8\pi\phi^2|}} \right) + \frac{(1+3\lambda)\nu + 8\pi\phi^2}{\lambda|\nu + 8\pi\phi^2|}. \quad (3.10)$$

This expression will be used for the investigation of the acceleration/deceleration intervals at early times. Accordingly, for the Brans-Dicke theory, equation (2.22) takes the form $\frac{\ddot{a}}{a} = -H^2 - \frac{k}{a^2} - \frac{\omega}{6} \frac{\dot{\phi}^2}{\phi^2}$, thus, for $k \geq 0$ and $\omega > 0$ there is no acceleration. In our model, the existence of the parameter ν gives the freedom to have acceleration without the need of introducing a self-coupling of the scalar field or through other mechanisms. Finally, the Ricci scalar R is found from equations (3.9) and (3.1) to have the simple expression

$$R = -\frac{8\pi c^2}{\lambda a^6} \frac{(1+3\lambda)\nu + 4\pi(2-3\lambda)\phi^2}{(\nu + 8\pi\phi^2)^2}. \quad (3.11)$$

Next, we will discuss the four cases resulting from equation (3.6).

- Case I: If $\nu + 8\pi\phi^2 > 0$ and $\epsilon\lambda > 0$, then $\lambda\phi > 0$. Equation (3.6) is integrated to

$$a^2 = \frac{2c^2}{|\lambda|\rho_*} \frac{1}{\sqrt{\nu + 8\pi\phi^2}} \frac{\sigma \left| 4\pi\phi + \sqrt{2\pi} \sqrt{\nu + 8\pi\phi^2} \right|^{\pm \sqrt{\frac{2}{3|\lambda|}}}}{\left[1 - \sigma \left| 4\pi\phi + \sqrt{2\pi} \sqrt{\nu + 8\pi\phi^2} \right|^{\pm \sqrt{\frac{2}{3|\lambda|}}} \right]^2}, \quad (3.12)$$

where $\sigma > 0$ is integration constant and it should be $\sigma \left| 4\pi\phi + \sqrt{2\pi} \sqrt{\nu + 8\pi\phi^2} \right|^{\pm \sqrt{\frac{2}{3|\lambda|}}} < 1$. We study next the behaviour of this solution for $\phi > 0$, which implies $\epsilon, \lambda > 0$.

For $\nu > 0$ the upper branch is defined for $0 < \phi < \phi_1 = \frac{1}{8\pi} (\sigma^{-\sqrt{\frac{3\lambda}{2}}} - 2\pi\nu\sigma\sqrt{\frac{3\lambda}{2}})$ and note that $2\pi\nu < \sigma^{-\sqrt{6\lambda}}$. At the minimum value $\phi = 0$, the scale factor a of the solution (3.12) gets a constant value with $\rho = 0$, thus the Ricci scalar (3.11) is also finite. At the maximum value $\phi = \phi_1$ the scale factor becomes infinite and again $\rho \rightarrow 0$. Although the quadrature (3.7) of the time cannot be integrated explicitly in terms of elementary functions, we can find the leading behaviour close to the minimum value of ϕ . It is seen that for $\phi = 0$ the time $t = 0$. Since at the minimum a, ϕ increases, we must have from (3.1) that $c > 0$, and thus the function $\phi(t)$ is increasing and it is like a good time parameter. Therefore, we have a universe which emerges at zero cosmic time at a finite volume and avoids the cosmological singularity both in density and curvature.

Equally importantly, there are parameters such that there is a transient accelerating era in the radiation epoch. For example, setting $\lambda = 2, \nu = 0.5, \sigma = 0.1, c = 3, \rho_* = 1$ in equation (3.10) and make a plot of $\frac{\ddot{a}}{a}$ we see that we get an initial decelerating era, followed by an accelerated period which finishes, and the universe enters again into deceleration. This transient acceleration era could be interpreted as an inflationary period with a graceful exit. It is also possible, depending on the parameters, to exist a temporary “breath” of the scale factor, so a during the expansion, for some period of time contracts, and then re-expands monotonically. This phenomenon might deserve further investigation. The corresponding Brans-Dicke solution arises from equation (3.12) for $\nu = 0$ and $\lambda > 0$. The result is that for $\lambda < \frac{2}{3}$ the above non-singular branch, where the universe emerges at finite radius, is lost here, and the solution becomes singular. For $\lambda > \frac{2}{3}$ ($\omega < 0$) Brans-Dicke possesses a bouncing solution. To give an estimate of the relative energy densities of the matter and the scalar field we refer that $\rho/\dot{\phi}^2 \rightarrow 0$ at the minimum scale factor and $\rho/\dot{\phi}^2 \rightarrow \infty$ at infinity.

For $\nu > 0$ the lower branch is defined for $\phi > \max\{0, \phi_2\}$, $\phi_2 = \frac{1}{8\pi} (\sigma\sqrt{\frac{3\lambda}{2}} - 2\pi\nu\sigma\sqrt{\frac{3\lambda}{2}})$, and it is always decelerating. For $2\pi\nu > \sigma\sqrt{6\lambda}$, the universe starts at zero a with infinite ϕ and infinite ρ, R , and finally tends to a constant scale factor for $\phi = 0, \rho = 0$. For $2\pi\nu < \sigma\sqrt{6\lambda}$, the universe starts at zero a with infinite ϕ and infinite ρ, R , and finally tends to infinite volume for $\phi = \phi_2, \rho = 0$. Therefore, these are typical singular solutions.

For $\nu < 0$ only the upper branch is valid which is defined for $\phi_3 < \phi < \phi_4$, where $\phi_3 = \sqrt{\frac{|\nu|}{8\pi}}$, $\phi_4 = \frac{1}{8\pi}(\sigma - \sqrt{\frac{3\lambda}{2}} + 2\pi|\nu|\sigma\sqrt{\frac{3\lambda}{2}})$ (it should also be $2\pi|\nu| < \sigma - \sqrt{6\lambda}$). At both ϕ_3, ϕ_4 the scale factor is infinite and $\rho \rightarrow 0$. Therefore, this solution describes a bouncing universe where the universe collapses from infinite volume, it has a bounce with all a, ρ and R finite, and re-expands to infinity.

- Case II: If $\nu + 8\pi\phi^2 < 0$ and $\epsilon\lambda < 0$, then $\lambda\phi > 0$. Equation (3.6) is integrated to

$$a^2 = \frac{c^2}{2|\lambda|\rho_*} \frac{1}{\sqrt{|\nu| - 8\pi\phi^2}} \left\{ 1 + \tan^2 \left[\sigma \pm \frac{1}{\sqrt{6|\lambda|}} \arcsin \left(\sqrt{\frac{8\pi}{|\nu|}} \phi \right) \right] \right\}, \quad (3.13)$$

where σ is an integration constant and it should be $0 < \sigma \pm \frac{1}{\sqrt{6|\lambda|}} \arcsin \left(\sqrt{\frac{8\pi}{|\nu|}} \phi \right) < \frac{\pi}{2}$. For $\phi > 0$, it is implied $\epsilon < 0, \lambda > 0$.

For the upper branch there are four cases, but they all have the same features. For $\sigma > 0$ and $0 < \frac{\pi}{2} - \sigma < \frac{\pi}{2\sqrt{6\lambda}}$ it is $0 < \phi < \phi_1$, $\phi_1 = \sqrt{\frac{|\nu|}{8\pi}} \sin \left[\sqrt{6\lambda} \left| \frac{\pi}{2} - \sigma \right| \right]$. At the minimum value $\phi = 0$, the scale factor a of the solution (3.13) gets a constant value with $\rho = 0$, R finite and $t = 0$. At the maximum value $\phi = \phi_1$ the scale factor becomes infinite and also $\rho \rightarrow 0$. Therefore, we have again a non-singular universe in all the quantities, volume, energy density and curvature. Analogously to the previous case, here there are parameters such that the evolution starts with acceleration which is followed by an entrance into deceleration (this happens for example for $\lambda = 0.1, \nu = -0.5, \sigma = 0.1, c = 3, \rho_* = 1$). The present solution has some additional interest since it occurs for very small values of λ making $\omega > 0$. In addition, it is valid for negative values of ν which will be seen in the next section that they provide the correct phenomenology at late times; therefore, we can have a unified picture for all times with a unique mechanism of energy transfer between matter and the scalar field. For $-\frac{\pi}{2\sqrt{6\lambda}} < \sigma < 0$ and $\frac{\pi}{2} - \sigma > \frac{\pi}{2\sqrt{6\lambda}}$, it is $\phi_2 < \phi < \phi_3$, where $\phi_2 = \sqrt{\frac{|\nu|}{8\pi}} \sin(|\sigma|\sqrt{6\lambda})$, $\phi_3 = \sqrt{\frac{|\nu|}{8\pi}}$. For $\sigma > 0$ and $\frac{\pi}{2} - \sigma > \frac{\pi}{2\sqrt{6\lambda}}$, it is $0 < \phi < \phi_3$. For $\sigma < 0$ and $\frac{\pi}{2} - \sigma < \frac{\pi}{2\sqrt{6\lambda}}$, it is $\phi_2 < \phi < \phi_1$. In all these cases the behaviour is the same with the appearance of a finite universe expanding to infinity.

For the lower branch there are also four cases. For $0 < \sigma < \frac{\pi}{2}$ and $\sigma < \frac{\pi}{2\sqrt{6\lambda}}$, it is $0 < \phi < \phi_2$ and the universe starts non-singular and ends at a finite scale factor. For $0 < \sigma - \frac{\pi}{2} < \frac{\pi}{2\sqrt{6\lambda}}$ and $\sigma > \frac{\pi}{2\sqrt{6\lambda}}$, it is $\phi_1 < \phi < \phi_3$ and the solution represents a bouncing universe. Finally, for $\frac{\pi}{2\sqrt{6\lambda}} < \sigma < \frac{\pi}{2}$ where it is $0 < \phi < \phi_3$, or for $\frac{\pi}{2} < \sigma < \frac{\pi}{2\sqrt{6\lambda}}$ where $\phi_1 < \phi < \phi_2$, we have a non-singular universe extending to infinity.

- Case III: If $\nu + 8\pi\phi^2 > 0$ and $\epsilon\lambda < 0$, then $\lambda\phi < 0$. Equation (3.6) is integrated to

$$a^2 = \frac{c^2}{2|\lambda|\rho_*} \frac{1}{\sqrt{\nu + 8\pi\phi^2}} \left[1 + \tan^2 \left(\sigma \pm \frac{1}{\sqrt{6|\lambda|}} \ln \left| 4\pi\phi + \sqrt{2\pi} \sqrt{\nu + 8\pi\phi^2} \right| \right) \right], \quad (3.14)$$

where σ is integration constant and it should be $0 < \sigma \pm \frac{1}{\sqrt{6|\lambda|}} \ln \left| 4\pi\phi + \sqrt{2\pi} \sqrt{\nu + 8\pi\phi^2} \right| < \frac{\pi}{2}$. For $\phi > 0$, it is implied $\epsilon > 0, \lambda < 0$.

For $\nu > 0$ the upper branch possesses two cases. The first case is characterized by the conditions $\phi_1 < 0$ and $\phi_2 > 0$, where $\phi_1 = \frac{1}{8\pi}(e^{-\sigma\sqrt{6|\lambda|}} - 2\pi\nu e^{\sigma\sqrt{6|\lambda|}})$, $\phi_2 = \frac{1}{8\pi}[e^{\sqrt{6|\lambda|}(\frac{\pi}{2}-\sigma)} - 2\pi\nu e^{-\sqrt{6|\lambda|}(\frac{\pi}{2}-\sigma)}]$, which means equivalently $e^{-2\sigma\sqrt{6|\lambda|}} < 2\pi\nu < e^{2\sqrt{6|\lambda|}(\frac{\pi}{2}-\sigma)}$. Then it is $0 < \phi < \phi_2$. At the minimum value $\phi = 0$ the universe is non-singular, with finite a , $\rho = 0$ and R finite. At the maximum value $\phi = \phi_2$ the scale factor extends to infinity with $\rho \rightarrow 0$. Moreover, there are parameters such that the universe starts decelerating, there is a transient accelerating era and an exit to deceleration (for example this happens for $\lambda = -0.5, \nu = 0.5, \sigma = 0.1, c = 3, \rho_* = 1$). The second case refers to $0 < \phi_1 < \phi_2$ and it is $\phi_1 < \phi < \phi_2$. Again at the minimum $\phi = \phi_1$ all a, ρ and R are finite, while at the maximum $\phi = \phi_2$, the volume becomes infinite.

For $\nu > 0$ the lower branch has also two cases. For the first it is $\phi_4 < 0, \phi_3 > 0$, where $\phi_3 = \frac{1}{8\pi}(e^{\sigma\sqrt{6|\lambda|}} - 2\pi\nu e^{-\sigma\sqrt{6|\lambda|}})$, $\phi_4 = \frac{1}{8\pi}[e^{\sqrt{6|\lambda|}(\sigma-\frac{\pi}{2})} - 2\pi\nu e^{-\sqrt{6|\lambda|}(\sigma-\frac{\pi}{2})}]$ and thus $0 < \phi < \phi_3$. At the minimum $\phi = 0$ the universe starts non-singular and ends for $\phi = \phi_3$ also to a finite universe. For the second case it is $0 < \phi_4 < \phi_3$ and thus $\phi_4 < \phi < \phi_3$. Again we have a non-singular universe extending to infinity.

For $\nu < 0$ there is a multiplicity of cases for each branch and we get analogous behaviours as those mentioned above.

- Case IV: If $\nu + 8\pi\phi^2 < 0$ and $\epsilon\lambda > 0$, then $\lambda\phi < 0$. Equation (3.6) is integrated to

$$a^2 = \frac{c^2}{2|\lambda|\rho_*} \frac{1}{\sqrt{|\nu| - 8\pi\phi^2}} \sinh^{-2} \left[\sigma \mp \frac{1}{\sqrt{6|\lambda|}} \arcsin \left(\sqrt{\frac{8\pi}{|\nu|}} \phi \right) \right], \quad (3.15)$$

where σ is integration constant and it should be $\sigma \mp \frac{1}{\sqrt{6|\lambda|}} \arcsin \left(\sqrt{\frac{8\pi}{|\nu|}} \phi \right) > 0$. For $\phi > 0$, it is implied $\epsilon < 0$, $\lambda < 0$.

For the upper branch it should be $\sigma > 0$ and $0 < \phi < \phi_1$, where $\phi_1 = \sqrt{\frac{|\nu|}{8\pi}} \sin(|\sigma|\sqrt{6|\lambda|})$. At the minimum $\phi = 0$ we get a non-singular universe with a finite, $\rho = 0$ and R finite. At the maximum $\phi = \phi_1$ the scale factor extends to infinity and $\rho \rightarrow 0$. We can easily get decelerating solutions in this branch.

For the lower branch there are two cases. The first one has $\sigma > 0$ and $0 < \phi < \phi_2$, where $\phi_2 = \sqrt{\frac{|\nu|}{8\pi}}$. For $\phi = 0$ the universe is non-singular with finite all a, ρ and R , while for $\phi = \phi_2$ the scale factor goes to infinity with $\rho \rightarrow 0$. We can check numerically that there is an intermediate contracting phase. Initially we can have a decelerating phase, followed by an eternal acceleration. The second case has $\sigma < 0$ and $\phi_1 < \phi < \phi_2$. At both ϕ_1, ϕ_2 the scale factor $a \rightarrow \infty$ and the solution represents a bouncing universe.

To resume with the most interesting solutions, for $\nu > 0$ (with either $\lambda > 0$ - Case I or $\lambda < 0$ - Case III) we can have an avoidance of the initial singularity (in all, scale factor, proper time, energy density and curvature) and at the same time a transient accelerating period within deceleration in the radiation regime. For $\nu < 0$ (such that $\nu + 8\pi\phi^2 < 0$) and $\lambda > 0$ - Case II we can have again a non-singular universe in all the quantities, where the evolution starts with acceleration which is followed by an entrance into deceleration.

In all these cases, due to the interaction term on the right-hand side of equation (2.14), we have an entropy production which could help to confront the cosmological entropy problem. Namely, we make use of the standard thermodynamic relation $dU + pdV = TdS$, where $U = \rho V$ is the energy contained in a comoving volume $V \propto a^3$ with corresponding entropy S and temperature T . Then, equation (2.14) can be written as

$$\frac{T}{V} \dot{S} = \frac{\nu}{\phi(\nu + 8\pi\phi^2)} \rho \dot{\phi}. \quad (3.16)$$

Whenever the right-hand side of equation (3.16) is positive, as it happens with the branches mentioned above, the universe evolution leaves adiabaticity and leads to entropy production. This entropy is shared initially between all relativistic species (photons, baryons, etc.). But as the universe cools down, the massive particles freeze out and the entropy is only shared to the photons. These photons propagate in the universe and are observed today with their high value of entropy per baryon, while the corresponding temperature T scales as $1/a$. Of course there is the constraint that the mechanism which produces entropy and matter, should not create too much matter, in order to comply with observations.

IV. LATE-TIMES COSMOLOGICAL EVOLUTION

In this section we will investigate the late-times cosmology of the generalized Brans-Dicke gravity. The Friedmann equations (2.11) and (2.12) can be written in a more familiar form

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3\phi} (\rho + \rho_{DE}) \quad (4.1)$$

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -\frac{8\pi}{\phi} (p + p_{DE}), \quad (4.2)$$

where we have defined the effective dark energy and effective dark pressure as

$$\rho_{DE} \equiv -\frac{3\phi^2}{\nu + 8\pi\phi^2} H \dot{\phi} + \frac{\phi}{2\lambda} \frac{\nu + 4\pi(2 - 3\lambda)\phi^2}{(\nu + 8\pi\phi^2)^2} \dot{\phi}^2 \quad (4.3)$$

$$p_{DE} \equiv \frac{\phi}{2\lambda} \frac{(1 + 2\lambda)\nu + 4\pi(2 - 3\lambda)\phi^2}{(\nu + 8\pi\phi^2)^2} \dot{\phi}^2 + \frac{\phi^2}{\nu + 8\pi\phi^2} (2H\dot{\phi} + \ddot{\phi}). \quad (4.4)$$

Observe that we recover the standard Brans-Dicke cosmology if $\nu = 0$. If $\nu \neq 0$ the Friedmann equations (4.1) and (4.2) incorporate in a non-trivial way the time evolution of the scalar-field which, as we will discuss in the following, brings new features in the late-times cosmological evolution compared to the standard Brans-Dicke theory.

To study the late-times cosmology we define the equation-of-state parameter for the effective dark energy sector

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} , \quad (4.5)$$

and we introduce the deceleration parameter through

$$q = -1 - \frac{\dot{H}}{H^2} . \quad (4.6)$$

Then, according to (4.1), we can define the density parameters as

$$\Omega_m = \frac{8\pi\rho}{3\phi H^2} , \quad \Omega_{DE} = \frac{8\pi\rho_{DE}}{3\phi H^2} . \quad (4.7)$$

Thus, the deceleration parameter (4.6) can be written for $k = 0$ in terms of the density parameters as

$$q = \frac{1}{2} + \frac{3}{2}(w_m\Omega_m + w_{DE}\Omega_{DE}) , \quad (4.8)$$

with $w_m \equiv p/\rho$ the matter equation-of-state parameter.

If we combine equations (2.6) and (2.9), we get the conservation equation of ρ_{DE}

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = \left(\frac{1}{\phi}\rho_{DE} + \frac{8\pi\phi}{\nu + 8\pi\phi^2}\rho \right) \dot{\phi} . \quad (4.9)$$

Additionally, we can rewrite the conservation equations (2.14) and (4.9) in the form

$$\left(\frac{1}{\phi} \rho \right) \dot{} + \frac{3H}{\phi}(\rho + p) = -\frac{8\pi}{\nu + 8\pi\phi^2}\rho\dot{\phi} = -Q \quad (4.10)$$

$$\left(\frac{1}{\phi} \rho_{DE} \right) \dot{} + \frac{3H}{\phi}(\rho_{DE} + p_{DE}) = \frac{8\pi}{\nu + 8\pi\phi^2}\rho\dot{\phi} = Q . \quad (4.11)$$

The above relations (4.10) and (4.11) resemble the standard relations

$$\begin{aligned} \dot{\rho} + 3H(\rho + p) &= -Q \\ \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) &= Q , \end{aligned} \quad (4.12)$$

which give rise to an interaction between the effective dark energy and the dark matter sector, with the quantity Q giving the strength and the form of this interaction. However, there are two main differences. The first one is that the scalar field ϕ , which plays the role of a varying gravitational constant, enters the variation too and hence, the quantity that is strictly conserved is $(\rho + \rho_{DE})/\phi$. The second one is that the above interaction, and the specific expression for Q , is determined by the theory itself through the consistency of the extension of the standard Brans-Dicke theory.

In the existing literature, the interaction term Q is only phenomenological and it is not resulted from a consistent field theoretic model. In [46], for example, this coupling parameterizes the interaction between dark matter to a quintessence field and it is constrained from observations compared to the couplings of the fields to gravity. In [47] an interactive field theory is discussed in which the interaction Q describes the coupling of a fermionic field for dark matter to a bosonic field for dark energy. In the model discussed here, this interactive term results from the theory, and as can be seen from equation (4.10), for $\nu = 0$ it recovers the standard conservation equation (2.18) of non-interacting Brans-Dicke cosmology. The dependence of the interactive term Q on the parameter ν can be understood from the fact that from equation (2.9) or (2.14) the matter energy-momentum tensor is sourced by the scalar field, where the interaction is controlled by the parameter ν . Similarly, in equation (2.8) or (2.13) it is seen that the matter energy-momentum tensor is the source of the scalar field and the strength of the interaction is now controlled by λ .

The main motivation for the study of interacting models is to solve the coincidence problem with a suitable coupling between dark energy and dark matter and also to drive the transition from an early matter dominated era to a phase of accelerated expansion [9]. The important issue which is known is that there is the possibility for a scaling attractor to be accelerating only if the standard conservation of matter is violated. Another motivation is the prediction of the variation of w_{DE} during the evolution of the universe. The variation of w_{DE} was studied in [48] using an appropriate coupling between dark energy and dark matter and it was shown in a holographic model that a transition from $w_{DE} > -1$ to $w_{DE} < -1$ occurs, claiming that this property could serve as an observable feature of the interaction

between dark energy and dark matter, in addition to its influence on the small l Cosmic Microwave Background spectrum argued in [49].

Concerning equations (4.10), (4.11) there are strong constraints about the possible non-gravitational interactions of baryons. In most of the works on interacting dark energy models, the baryonic matter is separately conserved and only dark matter is allowed to interact non-gravitationally with dark energy. This would mean that the parameter ν corresponding to the conservation equation of the baryonic matter is restricted to very small values, so that the weak equivalence is respected by this component. In this work we do not consider such a distinction between the two components (dark/baryonic) since we do not attempt a fitting against real data and our analysis is merely indicative. Our main results, however, are not expected to change significantly under a more detailed analysis since the baryonic matter forms a small percentage of the total one.

To study the two Friedmann equations (4.1) and (4.2) we will ignore at first approximation the radiation component and consider that the universe is composed of non-relativistic matter ρ with negligible pressure $p \ll \rho$, thus we set $w_m = 0$. Even in this case, the complexity of equations (4.3) and (4.4) does not allow us to have analytical solutions, and hence in the following we will use numerical methods. In order to acquire a consistent cosmology, in agreement with observations, we restrict to the flat case, namely imposing $k = 0$, and we set the present values of the density parameters to $\Omega_{m0} \approx 0.3$ and $\Omega_{DE0} \approx 0.7$ [44]. In addition, we set the standard value $H_0 = 0.71 \times 10^{-10} \text{ yr}^{-1}$ and we can assume for convenience that the present scale factor is $a_0 = 1$ and the units are chosen so that $\phi_0 = 1$. These values allow us to determine from equation (4.3) and the second of equations (4.7) the present value $\dot{\phi}_0$ as a function of the parameters λ, ν .

Note that since the expression of ρ_{DE} in equation (4.3) is quadratic in $\dot{\phi}$, the above procedure will give two values for $\dot{\phi}_0$, and each one will give rise to a different cosmological evolution. Additionally, there can be regions of the model parameters λ, ν for which $\dot{\phi}_0$ becomes complex, and thus the corresponding cosmologies are not realistic. Next, combining equation (2.19) with the first of relations (4.7), and applying them on present time, we obtain (of course ϕ is positive)

$$\rho_* = \frac{3a_0^3 \Omega_{m0} H_0^2}{8\pi} \sqrt{|\nu + 8\pi\phi_0^2|}. \quad (4.13)$$

This equation provides the integration constant ρ_* defining the evolution of the energy density ρ . Having chosen the initial conditions $a_0, \phi_0, \dot{\phi}_0$ as above, we are going to solve numerically the system of equations (2.11) and (2.13) for $a(t), \phi(t)$ with the function ρ given by equation (2.19). After these solutions have been obtained, we can derive the phenomenological quantities $\rho_{DE}, \Omega_m, \Omega_{DE}, w_{DE}, q$ as functions of time or of the redshift z . Having satisfied the present cosmological data, we can explore the cosmological behaviour for various values of the model parameters ν and λ , requiring a realistic cosmology, namely a present dark energy equation-of-state parameter w_{DE0} around -1 and a matter domination at earlier times.

In Fig. 1 we present the cosmological evolution for a spatially flat universe for the parameter choice $\nu = -100$ and $\lambda = 10$. As independent variable we use the redshift $z = -1 + a_0/a$. In the upper graph we depict the evolution of the matter and dark energy density parameters, which their behaviour show an agreement with observations. In the middle graph we present the evolution of the dark energy equation-of-state parameter w_{DE} . Finally, in the lower graph we depict the evolution of the deceleration parameter q , where the passage from deceleration to acceleration at late times can be seen.

Note that in the above figures we have checked that $\phi(t)$ exhibits an increasing behaviour up to very large redshifts which is the result of the fact that we have chosen ν and λ in order for $\dot{\phi}_0$ to be positive. This means that the gravitational constant G is decreasing with time as it is expected to be more reasonable in Brans-Dicke theory. However, ϕ does not cross the zero value which would mean a discontinuity from $+\infty$ to $-\infty$ for G . Additionally, we have checked that $\frac{\dot{\phi}}{\phi H}|_0 \lesssim 10^{-2}$ which is necessary in order to be consistent with the bounds of the variation of the Newton's constant. The redshift at which acceleration starts, has been reasonably determined observationally to $z = 0.74 \pm 0.05$ [50], and might set non-redundant constraints on the parameter ν . The product of the age of the universe t_0 with the current Hubble constant H_0 is $0.81 \lesssim t_0 H_0 \lesssim 1.09$ [44], [51]-[53] and might also set constraints on the parameters. We will not consider these two bounds here.

As it is well known the standard Brans-Dicke theory cannot give a consistent cosmology, and in particular it cannot lead to acceleration for realistic values of ω . To remedy this, a potential for the scalar field was introduced which is beyond the original construction of the Brans-Dicke theory because this potential term drives the acceleration and not the Brans-Dicke kinetic terms. Also other alternatives were assumed, as discussed in the Introduction. In the extended Brans-Dicke model studied here, the late cosmological evolution is in agreement with observations and in particular the new parameter ν , which arises from the generalization of the standard Brans-Dicke theory, can drive alone the acceleration without the need of a potential.

Generalizing Fig. 1, there is a range of the parameters $\lambda > 0$ and ν sufficiently negative such that $\nu + 8\pi\phi^2 < 0$, which provide a late-times cosmological evolution in agreement with observations. Such values of λ, ν can match with

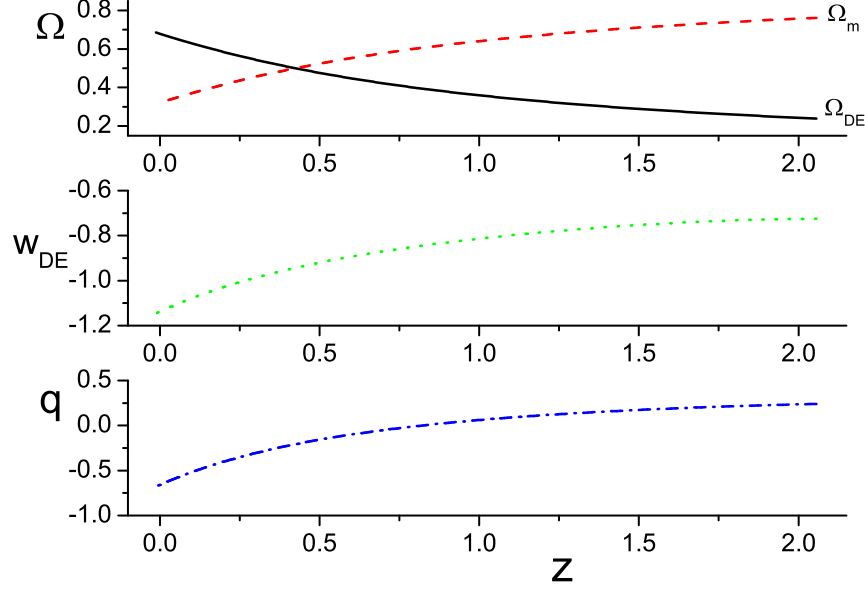


FIG. 1: The late-times cosmological evolution for a spatially flat universe, for the parameter choice $\nu = -100$ and $\lambda = 10$ in units where $\phi_0 = 1$, having imposed $\Omega_{m0} \approx 0.3$, $\Omega_{DE0} \approx 0.7$ at present, and having set the present scale factor $a_0 = 1$. As independent variable we use the redshift $z = -1 + a_0/a$. In the upper graph we depict the evolution of the matter and dark energy density parameters. In the middle graph we present the evolution of the dark energy equation-of-state parameter w_{DE} . In the lower graph we depict the evolution of the deceleration parameter q .

Case II of the early-times cosmic evolution discussed in (3.13). Thus for this parameter region, there is a unified description of the universe cosmic history to account for inflation, matter domination and late-times acceleration. In addition, from the conservation equation (2.14), since $\dot{\phi} > 0$, it is seen that $\dot{\rho}$, beyond the standard dilution, gets a positive contribution, which means that there is an energy transfer from the scalar field to the dark matter.

For completeness, we mention that one may also obtain cosmological evolution in agreement with observations, namely similar to Fig. 1, in the case where $\lambda < 0$, ν negative but not sufficiently, i.e. with $|\nu| \sim 8\pi\phi^2$ and $\nu + 8\pi\phi^2 > 0$. Now, there is an energy transfer from the dark matter sector to the dark energy component. On the other hand, for $\nu < 0$ with $|\nu| \ll 8\pi\phi^2$ (which is a slight deviation of Brans-Dicke), or for $\nu \geq 0$ (which includes the usual Brans-Dicke with $\nu = 0$) a cosmological evolution in agreement with observations is impossible (one might actually get acceleration but with increasing Ω_m).

One comment should be added at this point. As we have seen, for $\nu < 0$ consistent cosmologies can arise. However, for such ν 's, equations (2.11), (2.12), (2.14) present a pole for the value $\phi^2 = -\nu/(8\pi)$. It is reasonable to wonder if this divergence propagates any divergence in some quantity at finite time, such as the scale factor, Hubble parameter e.t.c. The answer is negative. For most of the parameter values there is no divergence in any quantity. This has been checked from the far future up to a high redshift $z = 3600$ where the validity of the dust solution terminates and the radiation solution starts. The reason for this smooth behaviour is that the scalar field is at all times away from the pole, so the pole lies outside the regime of applicability of the solution. We note however that, for some parameter regions, such as $\nu + 8\pi\phi^2 < 0$ with ϕ increasing, the pole value $\phi^2 = -\nu/(8\pi)$ might be reached in the future (unless if one chooses the initial conditions for ϕ so that it takes an asymptotically constant value away from the pole). In this case, the scale factor becomes infinite at a finite time, which is just the realization of a Big Rip (e.g. [54]). This is expected to happen in all models where super-acceleration is achieved and the dark energy equation-of-state parameter lies in the phantom regime.

In Fig. 2 we present the evolution of the deceleration parameter q for various values of ν and for fixed λ . We consider $\lambda < 0$ and the two regions: $\nu < 0$ with $\nu + 8\pi\phi^2 > 0$, and $\nu > 0$. As mentioned above, for intermediate negative values of ν we obtain late-times acceleration. However, for positive ν , as well as for $|\nu|$ very small, late-times acceleration cannot be obtained. This behaviour verifies the basic advantage of the present model that a non-zero parameter ν can lead to new and qualitatively different results that can make Brans-Dicke theory in agreement with

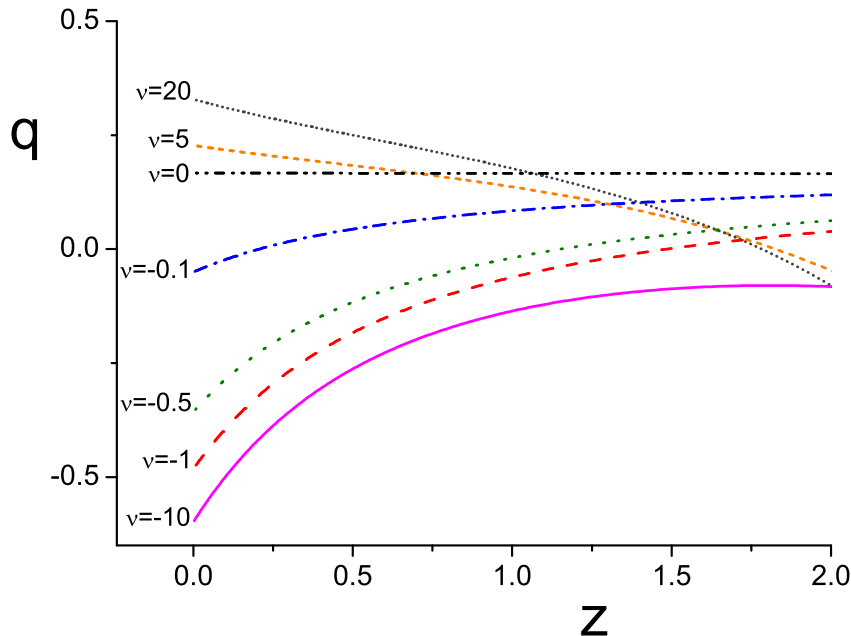


FIG. 2: The evolution of the deceleration parameter q versus the redshift z for a spatially flat universe, for $\lambda = -10$ and for various values of ν .

observations without the need of introducing an arbitrary potential or other ingredients.

In summary, in the extended Brans-Dicke theory discussed here, one obtains in cosmology an interaction between the dark matter and the effective dark energy sectors, which leads to a late-times cosmological evolution in agreement with observations.

V. CONCLUSIONS

We consider a completion of Brans-Dicke theory presented in [41], derived assuming that the scalar field follows a simple massless wave equation sourced by the trace of the matter energy-momentum tensor, while at the same time the scalar field energy-momentum tensor contains terms with two derivatives each. The conservation equation of matter is modified through a specific interaction with the scalar field determined by the theory and parametrized by a new dimensionfull parameter ν . For $\nu = 0$ the standard Brans-Dicke theory is recovered. In the present work we investigated the cosmological implications of this theory.

Considering a spatially homogeneous and isotropic geometry we derived the Friedmann equations of the theory in which non-trivial field kinetic terms appear similar to the Brans-Dicke gravity, but with not trivial ϕ dependence. Additionally, the standard conservation equation of a perfect fluid gets extra contribution from its interaction with the scalar field. This non-conservation equation can be integrated and the conventional dilution of the matter energy density gets corrected by a ϕ -dependent factor. Significant modifications of the energy density evolution occur when the scalar field gets values comparable or smaller than the new mass scale ν .

In the radiation era the system has been reduced to a first order differential equation for the scale factor as a function of the scalar field. This equation has been integrated and the time dependence comes through a quadrature. Among the solutions found, there are branches of solutions for a range of parameters which avoid the initial singularity (in all, scale factor, proper time, energy density and curvature) and at the same time they possess in the radiation regime either a transient accelerating period within deceleration, or the evolution emanates with acceleration which is followed by an entrance into deceleration. Additionally, due to the interaction term in the non-conservation equation of matter, these branches lead to an entropy production, which remains to be investigated if can answer the cosmological

entropy problem.

In the dust era, we have integrated the system numerically and found late-times acceleration in agreement with the correct behavior of the density parameters and the dark energy equation of state. Moreover, the scalar field ϕ appears only a slight variation over a large redshift interval, which leads to agreement with the strict bounds on the variation of the gravitational constant G . There are parameters of the model which provide a unified description of the universe history, namely account for inflation, matter domination and late-times acceleration under the same mechanism of energy transfer between matter and the scalar field. The problem of the nature of the attractors of the theory which is related to the coincidence problem needs a separate investigation, as well as the complete confrontation with real data.

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